Roll No. ..

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MAR-21-210002

B. Tech. EXAMINATION, March 2021

Semester I (CBCS)

ENGINEERING MATHEMATICS-I (A & B)

MA-101

Time : 3 Hours *Maximum Marks : 60*

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt *Five* questions in all, selecting *one* question from each Sections A, B, C and D. Q. No. 9 is compulsory.

Section A

1. (a) For what values of a and b do the equations :

$$x + 2y + 3z = 6$$
$$x + 3y + 5z = 9$$
$$2x + 5y + az = b$$

have

(i) no solution

(ii) a unique solution

(iii) more than one solution.

(b) Find the rank of the matrix :

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

2. (a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

(b) If λ be an eigen value of a non-singular matrix A, show that $\frac{|A|}{\lambda}$ is an eigen value of the matrix adj. A.

Section B

3. (a) Prove that $\frac{z^2 - 1}{z^2 + 1} = i \tan \theta$, where $z = e^{i\theta}$.

(b) If $\sin(A+iB)=x+iy$, prove that :

(i) $$\frac{x^2}{\cosh^2 B}+\frac{y^2}{\sinh^2 B}=1$$ 8

(ii) $$x^2\operatorname{cosec}^2 A-y^2\sec^2 A=1.$$

4. (a) Find all values of z such that $\sin z = 0$.

(b) If $u-v=(x-y)(x^2+4xyz+y^2)$ and $f(z)=u+iv$ is an analytic function of $z=x+iy$, find $f(z)$ in terms of z.

Section C

5. (a) If $y=\sin(m\sin^{-1}x)$, prove that :

$$(1-x^2)y_{n+2}=(2n+1)xy_{n+1}+(n^2-m^2)y_n$$

(b) Find $\frac{dy}{dx}$, when $(\cos x)^y=(\sin y)^x$.

6. (a) Evaluate $\int_0^1\int_x^{\sqrt{2-x^2}}\frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$ by changing the order of integration.

(b) Prove that :

$$\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta\times\int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}}=\pi$$

Section D

7. (a) If $r=|\vec{r}|$, where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, prove that :

$$\text{grad}\left(\frac{1}{r^2}\right)=-\frac{2\vec{r}}{r^4}$$

(b) Find the values of a, b, c for which the vector $\vec{V}=(x+y+az)\hat{i}+(bx+3y-z)\hat{j}+(3x+cy+z)\hat{k}$ is irrotational.

8. Verify Stokes Theorem for the vector field $\vec{F}=(2x-y)\hat{i}-yz^2\hat{j}-y^2z\hat{k}$ over the upper half surface of $x^2+y^2+z^2=1$ bounded by its projection on the xy-plane.

(Compulsory Question)

9. (a) Find the sum of eigen values of the inverse of the matrix $A=\begin{bmatrix}3&0&0\\8&4&0\\6&2&5\end{bmatrix}$. 12

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(b) Prove that e^z is a periodic function, where $z = x + iy$ is a complex variable.

(c) Separate into real and imaginary parts of $\sinh(x + iy)$.

(d) If $u = \tan^{-1}\frac{(x^3 + y^3)}{x - y}$ prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

(e) Expand $\sinh x$ in terms of Maclaurin's series.

(f) Evaluate the double integral $\int_0^1 \int_0^x e^{y/x}\, dy dx$.

(g) Prove that B(m, n) = B(n, m).

(h) Prove that if $\vec{F}(t)$ has a constant magnitude, then $\vec{F}.\frac{d\vec{F}}{dt} = \vec{0}$.

(i) In what direction from (3, 1, –2) is the directional derivative of $\phi = x^2y^2z^4$ maximum and what is its magnitude.

(j) Use divergence theorem to show that $\oint \nabla r^2 . d\vec{S} = 6V$, where S is any closed surface enclosing a volume V.